QUESTION 1 The blanks below will be filled by students. (Except the score)

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For the solution of this question please use only the front face and if necessary the back face of this page.

[15 pt] a) Calculate

i) $\lim_{x\to 1} \frac{x-1}{\sqrt[3]{\mathbf{x}}-1}$ ii) $\lim_{x\to 1} \frac{\sin(\sqrt{x}-1)}{x-1}$ iii) $\lim_{x\to\infty}\left(\sqrt{2x+5}-\sqrt{3x+7}\right)$

(Do not use L'Hopital rule)

[10 pt] b) Find the domain of

$$y = \frac{\sqrt{x^2+x-2}}{\sqrt{-x^2+x+2}}.$$

a) i) $\lim_{x\to 1} \frac{x-1}{\sqrt[3]{x}-1} = \frac{0}{0} = \lim_{x\to 1} \frac{\cancel{(\sqrt[3]{x}-1)}(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{\cancel{\sqrt[3]{x}-1}} = \boxed{3}$

where $x-1 = (\sqrt[3]{x})^3 - 1^3 = (\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)$.

ii) $\lim_{x\to 1} \frac{\sin(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{0}{0} = \lim_{x\to 1} \underbrace{\frac{\sin(\sqrt{x}-1)}{\sqrt{x}-1}}_{=1} \cdot \underbrace{\lim_{x\to 1}\frac{1}{\sqrt{x}+1}}_{\frac{1}{2}} = \boxed{\frac{1}{2}}$

iii) $\lim_{x\to\infty}\left(\sqrt{2x+5}-\sqrt{3x+7}\right) = \infty - \infty$

$= \lim_{x\to\infty} \frac{(\sqrt{2x+5}-\sqrt{3x+7})(\sqrt{2x+5}+\sqrt{3x+7})}{\sqrt{2x+5}+\sqrt{3x+7}} = \lim_{x\to\infty}\frac{2x+5-3x-7}{\sqrt{2x+5}+\sqrt{3x+7}}$

$= \lim_{x\to\infty}\frac{-x-2}{\sqrt{2x+5}+\sqrt{3x+7}} = \boxed{-\infty}$

b)

$-x^2+x+2 = (2-x)(x+1) = 0$ at $\boxed{x=-1}$ and $\boxed{x=2}$

$x^2+x-2 = (x+2)(x-1) = 0$ at $\boxed{x=-2}$ $\boxed{x=1}$

x		-2		-1		1		2	
$-x^2+x+2$	−	−	−	0	+	+	+	0	−
x^2+x-2	+	0	−	−	−	0	+	+	+

y gives a real value for $x^2+x-2 \geq 0$ and $-x^2+x+2 > 0$

Then

D: $[1, 2)$

MATHEMATICS 101E | MIDTERM | 9 July 2012

QUESTION 2 The blanks below will be filled by students. (Except the score)

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[15 pt] a) Show that $(\mathbf{sinx})^2 + \mathbf{3x} - \mathbf{1} = \mathbf{0}$ has exactly one root.

[10 pt] b) Classify the discontinuities of $\mathbf{y} = ((\mathbf{x^2} - \mathbf{1})\mathbf{sinx})/(|\mathbf{x} - \mathbf{1}|\mathbf{x})$.

a) $f(x) = \sin^2 x + 3x - 1 = 0$ is continuous for all $x \in \mathbb{R}$.

$\lim_{x \to -\infty} f(x) = -\infty < 0$

$\lim_{x \to +\infty} f(x) = \infty > 0$

According to the intermediate theorem, $f(x)$ takes on "0" at least one number $c \in \mathbb{R}$.

On the other hand; $f'(x) = 2\sin x \cos x + 3$

$= \underbrace{\sin 2x}_{\in [-1,1]} + 3$

$2 \leq f'(x) \leq 4$ or; $f'(x) > 0$ then $f(x)$ increases on the interval $(-\infty, \infty)$. Thus; The root "c" is unique.

b) $y = \dfrac{(x^2-1)\sin x}{|x-1|\,x} = \begin{cases} \dfrac{(x+1)\sin x}{x} & \text{when } x > 1 \\ -\dfrac{(x+1)\sin x}{x} & \text{when } x < 1 \\ \frac{0}{0} : \text{undef.} & \text{when } x = 1 \end{cases}$

$x = 0$: $\lim_{x \to 0^\mp} f(x) = \lim_{x \to 0^\mp} \dfrac{-(x+1)\sin x}{x} = -1$, $f(0)$: undefined.

$f(x)$ has a removable disc. at $\boxed{x=0}$

$x = 1$: $\lim_{x \to 1^+} f(x) = 2\sin 1$, $\lim_{x \to 1^-} f(x) = -2\sin 1$ $\Big\}$ One sided limits exist but differ. Then $f(x)$ has a jump discont. at $\boxed{x=1}$

MATHEMATICS 101E MIDTERM 9 July 2012

QUESTION 3 The blanks below will be filled by students. (Except the score)

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[15 pts] a) Suppose $\mathbf{f(x)}$ is a differentiable function that satisfies the equation

$$f(x+y) = f(x) + f(y) + x^2y + xy^2$$

for all real numbers $\mathbf{x}$ and $\mathbf{y}$. Suppose also that $\lim\limits_{\mathbf{x}\to 0} \dfrac{\mathbf{f(x)}}{\mathbf{x}} = \mathbf{1}$.

i) Compute $\mathbf{f(0)}$.

ii) Find $\mathbf{f}'(\mathbf{0})$ by using definition.

iii) Find $\mathbf{f}'(\mathbf{x})$ by using definition.

[10 pts] b) Find the tangent line to the parametric curve given by

$$x(t) = \cos(2t + \pi \tan t), \qquad y(t) = \sec^2(t)$$

at $\mathbf{t} = \dfrac{\pi}{4}$.

a) i) Let $x = y = 0$

$f(\cancel{0}) = f(\cancel{0}) + f(0) + 0 + 0 \Rightarrow \boxed{f(0) = 0}$

ii) $f'(0) = \lim\limits_{h\to 0} \dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h\to 0} \dfrac{f(h)}{h} = 1, \ \boxed{f'(0) = 1}$

iii) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{f(x)} + f(h) + x^2h + xh^2 - \cancel{f(x)}}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(h) + xh(x+h)}{h} = \underbrace{\lim\limits_{h\to 0} \dfrac{f(h)}{h}}_{=1} + \underbrace{\lim\limits_{h\to 0} x(x+h)}_{=x^2}$

$= 1 + x^2$

$\Rightarrow \boxed{f'(x) = 1 + x^2}$

b) $y' = \dfrac{dy/dt}{dx/dt} = \dfrac{2\sec t \cdot \sec t \cdot \tan t}{-\sin(2t + \pi \tan t) \cdot (2 + \pi \sec^2 t)}$

$m = y'\Big|_{t=\pi/4} = \dfrac{4}{(2+\pi \cdot 2)} = \dfrac{2}{1+\pi}, \quad x\left(\frac{\pi}{4}\right) = 0, \quad y\left(\frac{\pi}{4}\right) = 2$

$\dfrac{y-2}{x-0} = \dfrac{2}{1+\pi}$ or $\boxed{y = \dfrac{2}{1+\pi} + 2}$

MATHEMATICS 101E MIDTERM 9 July 2012

QUESTION 4 The blanks below will be filled by students. (Except the score)

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[25 pt] Sketch thre graph of the function

$$f(x) = \frac{x^2}{x^2 - 1}$$

by considering the domain of $\mathbf{f(x)}$, intercepts, continuity, symmetry, asymptotes, intervals on which the function is decreasing or incresing, exteremun and inflection points (if they exist) and concavity.

1) Domain $\mathbb{R} - \{+1, -1\}$.

2) Intercepts, $x=0 \Rightarrow \boxed{y=0}$

$y=0 \Rightarrow \boxed{x=0}$

3) Asymptotes:

$\lim\limits_{x\to\pm\infty} y = 1$, $\boxed{y=1}$ H.A, $\boxed{\text{NO OBLIGUE ASY.}}$

$\lim\limits_{x\to 1^+} \frac{x^2}{(x-1)(x+1)} = \frac{1}{0^+} = \infty$, $\lim\limits_{x\to 1^-} \frac{x^2}{(x-1)(x+1)} = -\infty$, $\boxed{x=1}$ V.A

$\lim\limits_{x\to -1^+} \frac{x^2}{(x-1)(x+1)} = -\infty$, $\lim\limits_{x\to -1^-} \frac{x^2}{(x-1)(x+1)} = \infty$, $\boxed{x=-1}$ V.A

4) Extrema:

$y' = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2} = 0$ at $\boxed{x=0} \in D$

; und. at $x = \pm 1 \notin D$

x		-1		0		1	
y'	+		+	0	-		-

f is increasing on $(-\infty,-1) \cup (-1,0)$
and decreasing on $(0,1) \cup (1,\infty)$
f(x) has a local max at x=0

5) Concavity:

$y'' = \frac{-2(x^2-1)^2 + 2x \cdot 2(x^2-1) \cdot 2x}{(x^2-1)^4} = \frac{\cancel{(x^2-1)}\left[-2x^2+2+8x^2\right]}{(x^2-1)^{\cancel{4}\,3}} = \frac{2(3x^2+1)}{(x^2-1)^3}$

$\neq 0$

; und. at $x = \pm 1 \notin D$.

no inflection

x		-1		1	
y''	+		-		+

The graph is concave up on $(-\infty,-1) \cup (1,\infty)$ and concave down on $(-1,1)$.

MATHEMATICS 101E MIDTERM 9 July 2012

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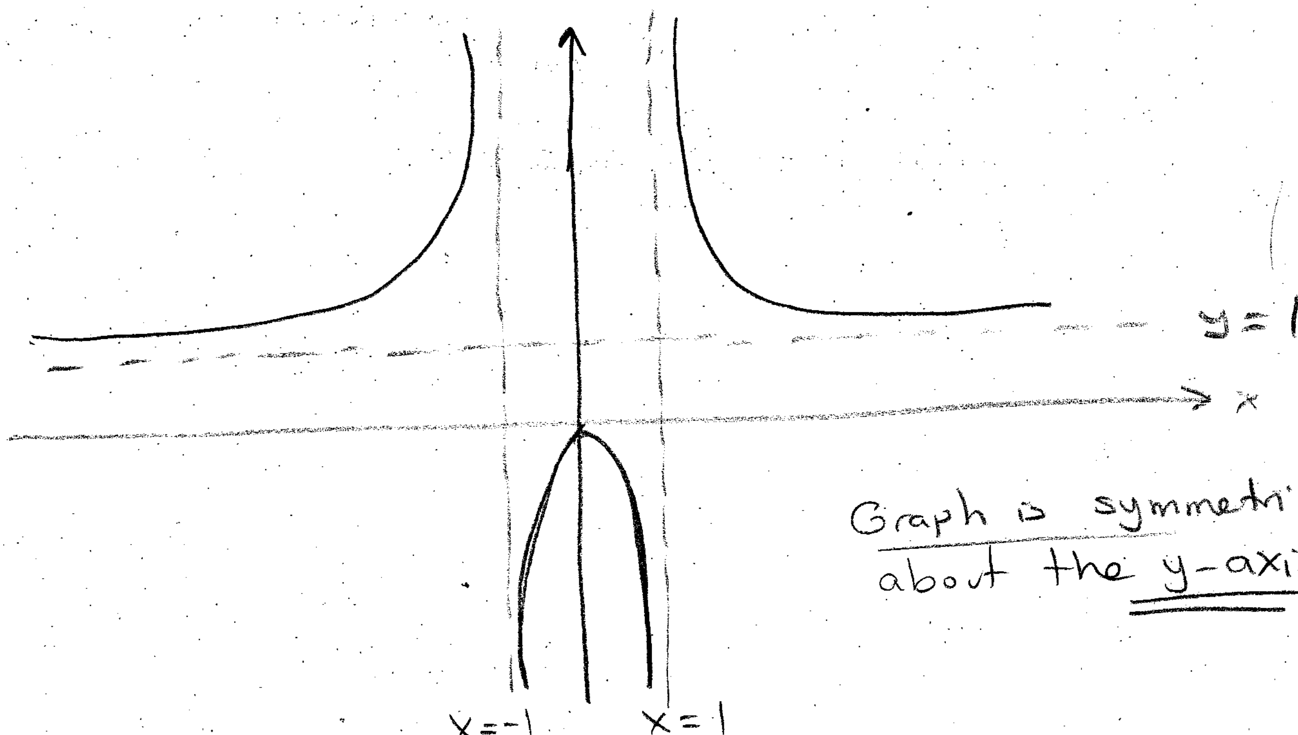
$$f(x) = \frac{x^2}{x^2 - 1}$$

by considering the domain of $\mathbf{f(x)}$, intercepts, continuity, symmetry, asymptotes, intervals on which the function is decreasing or incresing, exteremun and inflection points (if they exist) and concavity.

6)

x	$-\infty$		-1		0		1		∞
y'		$+$		$+$	0	$-$		$-$	
y''		$+$	$+\infty$ $-\infty$	$-$		$-$	$-\infty$ ∞	$+$	

H.A H.A



Graph is symmetric about the y-axis